

Noncommutative Open String Theories and Their Dualities

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The recently found non-critical open string theories is reviewed. These open strings, noncommutative open string theories (NCOS), arise as consistent quantum theories describing the low energy theory of D-branes in a background electric B-field in the critical limit. Focusing on the D3-brane case, we construct the most general (3+1) NCOS, which is described by four parameters. We study S and T -dualities of these theories and argue the existence of a U-duality group*.

1 Introduction and Review

It has been recently shown that noncommutative spaces, with the coordinates satisfying

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} , \quad (1.1)$$

arise as the worldvolume theory of Dp -branes in a $B_{\mu\nu}$ -field background (for a review see [1]). Also it has been shown that the low energy theory of such branes is a $NCU(1)$ SYM theory. Similar to the commutative case, when N number of such branes coincide this gauge symmetry is enhanced $NCU(n)$. For the $\theta^{0\mu} = 0$, the magnetic B-field, cases the large n limit of noncommutative gauge theories can be studied through a generalized form of gravity/gauge theory correspondence [2].

The case $\theta^{0\mu} \neq 0$, the electric B-field, seems to be more interesting because, *i*) as a field theory they are not unitary and hence not a well-defined quantum *field* theory [3, 4]. *ii*) for the 3+1 dimensional case, they appear as the strong coupling limit of $NCU(n)$ theory [5, 6]. The latter can be understood noting the Montonen-Olive duality and also the fact that the B-field background can be thought as the non-zero electric and magnetic background fields of the $U(n)$ gauge theory. Under the S-duality the electric and magnetic fields are replaced while the coupling goes to one over itself, i.e. in the strong coupling a magnetic noncommutative theory is mapped into an electric one, though in the critical electric field limit [5]. Intuitively, the open strings attached to D-branes+B-field are electric dipoles [7], and the critical E limit occurs when the tension of the open strings becomes equal to the force on this electric dipole [4].

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One can show that in this limit the closed strings of the bulk gravity theory become infinitely massive (compared to open strings) and hence they decouple from the dynamics of the theory. So in the end, in the critical E limit, we remain with a theory of noncommutative open string, NCOS, without any closed string states [4, 5]. The above argument is quite general and is true for Dp -branes, $p = 1, 2, \dots, 5$, however here we only focus on the $p = 3$ case. It is interesting to study the dualities, S and T, for this case.

In the next section we construct the most general 3+1 noncommutative open strings and then study the behaviour of these theories under the usual IIB S-duality transformation. We show that under S-duality in general a NCOS is mapped into another NCOS. However, there are some special cases for which a NCOS goes to a NCSYM [8].

In section 4, we compactify the 3+1 NCOS on a T^2_θ , so that the magnetic component of the B-field of our NCOS theory has a non-zero flux on the torus. We show that the NCOS, although being an open string theory without any closed strings, enjoys the full $SO(2, 2; Z) \simeq SL(2, Z) \times SL(2, Z)$ T-duality group. This T-duality group combined with the S-duality $SL(2, Z)$ group yield the usual U-duality group of closed strings on T^2 ; $SL(3, Z) \times SL(2, Z)$ [9]. We will argue that this U-duality group can be realized from the OM theory [10] picture by a T^3 compactification.

2 3+1 Noncommutative open strings, a systematic approach

Let us consider a D3 brane in the presence of a constant B -field with components B_{01} and B_{23} . Such constant B -field is equivalent to constant electric and magnetic vector fields on the brane pointing in the direction x^1 . In our conventions $\mu, \nu = 0, 1, 2, 3$, label the directions parallel to the brane, and x^a , $a, b = 4, \dots, 9$, the transverse directions. It is convenient to choose coordinates so that the closed string metric is of the form

$$\begin{aligned} g_{\mu\nu} &= \text{diag}(-\zeta, \zeta, \eta, \eta) , \\ g_{ab} &= K\delta_{ab} , \end{aligned} \tag{2.1}$$

where ζ , η and K are constant parameters which will be fixed later. The boundary conditions for the open string coordinates are given by

$$\partial_\sigma X^0 + E\partial_\tau X^1 \Big|_{\sigma=0,\pi} = 0 , \quad \partial_\sigma X^1 + E\partial_\tau X^0 \Big|_{\sigma=0,\pi} = 0 , \tag{2.2}$$

$$\partial_\sigma X^2 + B\partial_\tau X^3 \Big|_{\sigma=0,\pi} = 0 , \quad \partial_\sigma X^3 - B\partial_\tau X^2 \Big|_{\sigma=0,\pi} = 0 , \tag{2.3}$$

$$E \equiv B^0_1 = -\zeta^{-1}B_{01} , \quad B \equiv B^2_3 = \eta^{-1}B_{23} .$$

The transverse open string coordinates are free string coordinates obeying Dirichlet boundary conditions:

$$X^a \Big|_{\sigma=0,\pi} = x^a , \quad a = 4, \dots, 9 ,$$

where x^a denote the position in transverse space of N D3 branes. The solutions to the above boundary conditions are

$$\begin{aligned}
X^0 &= x^0 + 2\alpha'(p^0\tau - Ep^1\sigma) + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} [ia_n^0 \cos(n\sigma) - Ea_n^1 \sin(n\sigma)] , \\
X^1 &= x^1 + 2\alpha'(p^1\tau - Ep^0\sigma) + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} [ia_n^1 \cos(n\sigma) - Ea_n^0 \sin(n\sigma)] , \\
X^2 &= x^2 + 2\alpha'(p^2\tau - Bp^3\sigma) + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} [ia_n^2 \cos(n\sigma) - Ba_n^3 \sin(n\sigma)] , \\
X^3 &= x^3 + 2\alpha'(p^3\tau + Bp^2\sigma) + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} e^{-in\tau} [ia_n^3 \cos(n\sigma) + Ba_n^2 \sin(n\sigma)] , \\
X^a &= x^a + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^a}{n} e^{-in\tau} \sin(n\sigma) .
\end{aligned} \tag{2.4}$$

Now we fix the parameters ζ and η so that the *open string parameters* $G_{\mu\nu}$, $\theta_{\mu\nu}$ [1],

$$G_{\mu\nu} = g_{\mu\nu} - (Bg^{-1}B)_{\mu\nu} , \quad \theta^{\mu\nu} = 2\pi\alpha' \left(\frac{1}{g+B} \right)_A^{\mu\nu} , \tag{2.5}$$

get a simple form, convenient for describing the noncommutative open strings; i.e. we set

$$\zeta = (1 - E^2)^{-1} , \quad \eta = (1 + B^2)^{-1} , \tag{2.6}$$

so as to have

$$G_{\mu\nu} = \eta_{\mu\nu} , \quad \theta^{01} \equiv \theta = 2\pi\alpha' E , \quad \theta^{23} \equiv \tilde{\theta} = 2\pi\alpha' B . \tag{2.7}$$

Then the open string coupling is given by

$$G_s = g_s \left(\frac{\det G_{\mu\nu}}{\det(g_{\mu\nu} + B_{\mu\nu})} \right)^{\frac{1}{2}} = g_s \sqrt{(1 - E^2)(1 + B^2)} , \tag{2.8}$$

The other parameter in the metric, K , can be fixed independently. A suitable way for fixing it is using the D3-brane RR charge normalization. However, the way used in [5] is different. Demanding this RR charge to remain the same before and after the scaling of the closed string metric implies that

$$K = \left(\frac{\det(g_{ab})}{\det(g_{\mu\nu})} \right)^{1/2} = [(1 - E^2)(1 + B^2)]^{-1/5} . \tag{2.9}$$

The canonical commutation relations for the string coordinates then imply the following commutation relations for the mode operators:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} , \quad [x^\mu, p^\nu] = iG^{\mu\nu} , \quad [a_n^\mu, a_m^\nu] = n\delta_{n+m} G^{\mu\nu} , \quad [\alpha_n^a, \alpha_m^b] = n\delta_{n+m} K^{-1} \delta^{ab} .$$

One can also check that *the end points* of the string do not commute, i.e.

$$[X^\mu(\tau, 0), X^\nu(\tau, 0)] = i\theta^{\mu\nu} , \quad [X^\mu(\tau, \pi), X^\nu(\tau, \pi)] = -i\theta^{\mu\nu} .$$

The open string mass spectrum then becomes manifestly the same as the free string mass spectrum:

$$\alpha' M^2 = \alpha'(p_0^2 - p_1^2 - p_2^2 - p_3^2) = N - 1 .$$

The addition of fermions is as in the usual free open superstring theory (with the appropriate change in the normal ordering constant). The closed string mass spectrum is then given by

$$\alpha'(1 - E^2)(p_0^2 - p_1^2) - \alpha'(1 + B^2)(p_2^2 + p_3^2) = 2N + 2\bar{N} - 4 .$$

We see that as $E \rightarrow 1$ the energy of the closed string states goes to infinity, while the energy of open string excitations remains finite. In the limit $E \rightarrow 1$ with fixed B , α' and G_s , the closed string states are thus decoupled from the theory. Note that this limit requires $g_s \rightarrow \infty$ (see eq. (2.8)) and hence the closed string decoupling should be handled with more care; moreover one should still discuss why the massless closed string modes are also decoupled. This can be done noting that their effective coupling, the ten dimensional Newton constant,

$$G_N = \alpha'^4 g_s^2 K^{-5}(1 - E^2)(1 + B^2) = \alpha'^4 G_s^2(1 - E^2)(1 + B^2) ,$$

in the critical E limit goes to zero.

The resulting open string theory obtained in this limit contains the parameters G_s ,

$$\theta = 2\pi\alpha' , \quad \tilde{\theta} = 2\pi\alpha'B = \theta B . \quad (2.10)$$

Later we will show that one can also introduce another parameter χ associated with the RR scalar of type IIB theory. So, altogether our NCOS theory is defined by four parameters, α' , $\tilde{\theta}$, G_s and χ . The parameter $\alpha' = \theta/2\pi$ is the string scale and also characterizes the noncommutativity scale in the x^0 - x^1 directions, $\tilde{\theta}$ represents the noncommutativity scale in the x^2 - x^3 directions, G_s is the open string coupling, while χ is not relevant in the perturbative expansion.

General disc amplitudes for this "noncommutative" open string theory will have the form

$$\langle V(p^1) \dots V(p^N) \rangle_{\theta, \tilde{\theta}} = \exp \left[-\frac{i}{2} \sum_{n>m} p^n \wedge p^m \epsilon(\tau_n - \tau_m) \right] \langle V(p^1) \dots V(p^N) \rangle_{\text{free string}} , \quad (2.11)$$

where

$$p^n \wedge p^m = (p_0^n p_1^m - p_1^n p_0^m) \theta + (p_2^n p_3^m - p_3^n p_2^m) \tilde{\theta} .$$

In the case $B = 0$, one recovers the open string theory of [4, 5], obtained from open strings in a purely electric background.

It is worth noting that the noncommutative open strings also carry $U(N)$ Chan-Paton factors, and therefore besides the parameters $(\alpha', G_s, \tilde{\theta}, \chi)$ in order to specify a NCOS completely we should also determine N . We also note that in the $B \rightarrow \infty$, $\alpha' \rightarrow 0$, with

$$G_s = \text{fixed} , \quad \alpha' B = \text{fixed} , \quad (2.12)$$

limit of our NCOS theory we will recover the usual NCSYM theory, because $\alpha' \rightarrow 0$, massive open string excitations also decouple in this limit, and one is left with the Super Yang-Mills field theory

in noncommutative x^2 - x^3 space. Thus the present family of NCOS theories interpolates between the purely electric NCOS theory of [4, 5] and the NCSYM.

In general, the theory contains two energy scales, given by θ and $\tilde{\theta}$, or α' and $B\alpha'$. At distances L much larger than $\sqrt{\theta}$, $\sqrt{\tilde{\theta}}$, the theory reduces to ordinary SYM theory. For $B \gg 1$, there is a regime $\theta \ll L^2 < \tilde{\theta}$ in which the theory is SYM field theory on the noncommutative space x^2 - x^3 ; string effects can be ignored, but noncommutativity effects in x^2 - x^3 directions are important. If B is of order 1 or lower, then string effects appear at the same time as noncommutative effects.

3 $SL(2, Z)$ S-duality transformations on NCOS theory

Now we study the behavior of the open string theories of the previous section under type IIB $SL(2, Z)$ symmetry. Let us start with the corresponding type IIB supergravity solution. The Lorentzian supergravity solution representing a D3 brane in the presence of B_{01} and B_{23} fields is given in [8]. In the string frame, it is

$$\begin{aligned}
ds_{str}^2 &= f^{-1/2} \left[h'(-dx_0^2 + dx_1^2) + h(dx_2^2 + dx_3^2) \right] + f^{1/2} \left[dr^2 + r^2 d\Omega_5^2 \right], \quad (3.1) \\
f &= 1 + \frac{\alpha'^2 R^4}{r^4}, \quad h^{-1} = \sin^2 \alpha f^{-1} + \cos^2 \alpha, \quad h'^{-1} = -\sinh^2 \beta f^{-1} + \cosh^2 \beta, \\
B_{01} &= -\tanh \beta f^{-1} h', \quad B_{23} = \tan \alpha f^{-1} h, \\
e^{2\phi} &= g_s^2 h h', \quad \chi = \frac{1}{g_s} \sinh \beta \sin \alpha f^{-1} + \chi_0, \\
A_{01} &= \left(\frac{1}{g_s} \sin \alpha \cosh \beta + \chi_0 \tanh \beta \right) h' f^{-1}, \quad A_{23} = \left(\frac{1}{g_s} \sinh \beta \cos \alpha - \chi_0 \tan \alpha \right) h f^{-1}, \\
F_{0123u} &= \frac{1}{g_s} \cos \alpha \cosh \beta h h' \partial_r f^{-1}.
\end{aligned}$$

In the $r \rightarrow \infty$ region, the metric asymptotically approaches the Minkowski metric, and the asymptotic values for the different fields are as follows

$$(B^\infty)_1^0 = \tanh \beta \equiv E, \quad (B^\infty)_3^2 = \tan \alpha \equiv B, \quad (3.2)$$

$$(A^\infty)_1^0 = -\frac{1}{g_s} \frac{B}{\sqrt{1+B^2}} \frac{1}{\sqrt{1-E^2}} - \chi_0 E, \quad (A^\infty)_3^2 = \frac{1}{g_s} \frac{1}{\sqrt{1+B^2}} \frac{E}{\sqrt{1-E^2}} - \chi_0 B, \quad (3.3)$$

$$e^{2\phi_\infty} = g_s^2, \quad \chi_\infty = \frac{1}{g_s} \frac{B}{\sqrt{1+B^2}} \frac{E}{\sqrt{1-E^2}} + \chi_0. \quad (3.4)$$

We see that in the $E \rightarrow 1$ limit, with $G_s, \theta, \tilde{\theta}$ and χ_0 fixed (see eqs. (2.8), (2.10)), A_{01}^∞ and A_{23}^∞ also remain finite.

Under the $SL(2, Z)$ symmetry of the type IIB superstring the coupling

$$\lambda = \Theta + \frac{i}{g_s}, \quad \Theta \equiv \chi_\infty,$$

transforms as

$$\lambda \rightarrow \lambda' = \frac{a\lambda + b}{c\lambda + d}, \quad ad - bc = 1, \quad (3.5)$$

where a, b, c, d form an $SL(2, Z)$ matrix, whereas $B_{\mu\nu}$ and $A_{\mu\nu}$ (NSNS and RR) fields form a doublet:

$$\begin{pmatrix} B_{\mu\nu} \\ A_{\mu\nu} \end{pmatrix} \rightarrow \begin{pmatrix} B'_{\mu\nu} \\ A'_{\mu\nu} \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \begin{pmatrix} B_{\mu\nu} \\ A_{\mu\nu} \end{pmatrix}. \quad (3.6)$$

Therefore

$$g_s \rightarrow g'_s = g_s |c\lambda + d|^2. \quad (3.7)$$

The Einstein metric $g_{\mu\nu}^E = e^{-\phi/2} g_{\mu\nu}^{\text{str}}$ remains invariant, so the new string metric at $r = \infty$ is $|c\lambda + d| \eta_{\mu\nu}$. Using eqs. (3.3), (3.6), one finds that the transformed electric and magnetic fields are

$$\begin{aligned} E \rightarrow E' &= \frac{1}{|c\lambda + d|} [(d + c\Theta)E + c\frac{B}{G_s}(1 - E^2)], \\ B \rightarrow B' &= \frac{1}{|c\lambda + d|} [(d + c\Theta)B - c\frac{E}{G_s}(1 + B^2)]. \end{aligned} \quad (3.8)$$

Let us now consider the $E \rightarrow 1$ limit for the $SL(2)$ rotated parameters. In this limit the transformation (3.8) simplifies, since $g_s \rightarrow \infty$, $\lambda \rightarrow \Theta$. One can distinguish two different cases:

a) Irrational Θ :

In this case there are no integers c and d such that $c\lambda + d = 0$. In the $E \rightarrow 1$ and $g_s \rightarrow \infty$ limit, $|c\lambda + d|$ reduces to $|c\Theta + d|$ and E' and B' are:

$$E' = \frac{1}{|c\Theta + d|} (d + c\Theta)E = \pm 1, \quad B' = \pm B - \frac{c(1 + B^2)}{G_s |c\Theta + d|} = \text{finite}. \quad (3.9)$$

In the $E \rightarrow 1$ limit, the electric and magnetic fields are obtained by the simple transformation rules

$$\frac{1 - E^2}{|c\lambda + d|^2} \rightarrow \frac{1 - E'^2}{|c\Lambda + d|^2}, \quad \frac{1 + B^2}{|c\lambda + d|^2} \rightarrow \frac{1 + B'^2}{|c\Lambda + d|^2},$$

where

$$\Lambda = \chi_0 + \frac{i}{G_s}, \quad \chi_0 = \Theta - \frac{B}{G_s}. \quad (3.10)$$

The fact that $E'^2 \rightarrow 1$, with B' finite (and also $g'_s \sqrt{1 - E'^2}$ remains finite) shows that an $SL(2, Z)$ transformation leads to another NCOS with transformed parameters

$$(\theta, \tilde{\theta}, G_s, \Theta) \longrightarrow (\theta', \tilde{\theta}', G'_s, \Theta'),$$

where

$$\Theta' = \frac{a\Theta + b}{c\Theta + d}, \quad G'_s = G_s |c\Lambda + d|^2. \quad (3.11)$$

To find $\theta'^{\mu\nu}$ and $G'_{\mu\nu}$ we use eq.(2.5). One can choose coordinates (by scaling by suitable factors ζ and η given by (2.6) as in the previous section) so that the open string metric before the $SL(2)$ transformation is $G_{\mu\nu} = \eta_{\mu\nu}$. We find

$$G'_{\mu\nu} = \frac{|c\Lambda + d|^2}{|c\lambda + d|} \eta_{\mu\nu}, \quad (3.12)$$

$$\theta' = 2\pi\alpha' \frac{E'(1-E^2)}{|c\lambda + d|(1-E'^2)} = 2\pi\alpha' \frac{|c\lambda + d|}{|c\Lambda + d|^2} , \quad (3.13)$$

$$\tilde{\theta}' = 2\pi\alpha' \frac{B'(1+B^2)}{|c\lambda + d|(1+B'^2)} = 2\pi\alpha' B' \frac{|c\lambda + d|}{|c\Lambda + d|^2} . \quad (3.14)$$

Rescaling the coordinates so that the new open string metric $G'_{\mu\nu}$ is equal to $\eta_{\mu\nu}$, we conclude that the new NCOS theory has parameters,

$$\theta' = 2\pi\alpha' , \quad \tilde{\theta}' = 2\pi\alpha' B' . \quad (3.15)$$

b) Rational Θ :

In this case there exists an $SL(2, Z)$ transformation under which $c\Theta + d = 0$. The string coupling transforms as follows:

$$g_s \rightarrow g'_s = g_s |c\lambda + d|^2 = \frac{c^2}{g_s} , \quad (3.16)$$

i.e. this transformation relates strong and weak coupling regimes. From (3.8) one can find transformed E and B in the $E \rightarrow 1$ limit:

$$E' = \pm \frac{B\sqrt{1-E^2}}{\sqrt{1+B^2}} \rightarrow 0 , \quad B' = \pm \frac{\sqrt{1+B^2}}{\sqrt{1-E^2}} \rightarrow \pm\infty . \quad (3.17)$$

Therefore

$$\theta' = 2\pi\alpha' \frac{E'(1-E^2)}{|c\lambda + d|(1-E'^2)} = 0 , \quad \tilde{\theta}' = 2\pi\alpha' \frac{B'(1+B^2)}{|c\lambda + d|(1+B'^2)} = 2\pi\alpha' \frac{G_s}{c} = \text{finite} , \quad (3.18)$$

and the open string coupling is

$$G'_s = g'_s \sqrt{1-E'^2} \sqrt{1+B'^2} = g'_s B' = \frac{c^2(1+B^2)}{G_s} = \text{finite} . \quad (3.19)$$

In conclusion, for rational Θ there is an $SL(2, Z)$ transformation which maps the NCOS theory to a NCSYM field theory, with $\Theta = \frac{a}{c}$. In particular, we see that whenever $\Theta = 0$ the NCOS theory is S-dual to NCSYM theory, even if both E and B are non-vanishing. The reason is that in this case A_{01} given in (3.3) also vanishes, and therefore the S-dual theory will have a vanishing B_{01} . This generalizes the result of [5] that the theory with $E = 0$, $B \rightarrow \infty$ is S-dual to the theory with $E = 1$, $B = 0$. For irrational Θ , under $SL(2, Z)$ transformations the NCOS theory is always transformed to a NCOS theory with new parameters given by the above transformation rules.

4 NCOS Theory on a Torus and T-duality

Noncommutative super Yang-Mills theory on a general T^2 torus, unlike its commutative counterpart, enjoys the full $SO(2, 2; Z)$ T-duality group of the underlying string theory [1]. In the gauge theory language this is due to the ‘‘Morita equivalence’’, which is an equivalence for the gauge bundles on the noncommutative torus, with a proper mapping between the corresponding gauge groups and couplings, background magnetic fluxes, and volumes of the two tori.

NCOS theory (with rational χ) is equivalent to NCSYM theory by S-duality. On the other hand T-duality, although being a perturbative symmetry, should also hold in the strong coupling limit. Hence the T-duality group of NCOS theory with two spatial dimensions compactified on a 2-torus, must be same as the T-duality group of NCSYM theory, $SO(2, 2; Z)$.

Let us now consider T-duality transformations on NCOS theories we introduced previously. To study T-duality of NCOS theories is convenient to start with the appropriate type IIB configuration and then take a limit leading to a NCOS theory. Once the moduli parameters of the compactified theory are specified, it is easy to obtain the T-duality transformation properties. For the case of our interest, (3+1) dimensional NCOS with parameters, $(\alpha', G_s, \tilde{\theta}, \chi)$, we consider the compactification of the θ -plane x_2 - x_3 on a noncommutative torus two torus T_θ^2 . This theory can be realized as some particular limit of type IIB string theory in the presence of a $(D3, (F, D1))$ -brane system compactified on the two torus. We denote the complex and Kahler parameters of that torus by τ and ρ , respectively. Then the brane bound state is characterized by two integers m, N , whose ratio is proportional to the RR charge density corresponding to D-strings [11]. We choose coordinates so that the components of the closed string metric parallel to the brane bound state are $(-\frac{1}{1-E^2}, \frac{1}{1-E^2}, 1, 1)$. Along the lines of [7, 11], the spectrum of the open strings attached to the brane bound state is

$$\alpha' M^2 = \frac{|r + q\tau|^2}{\tau_2} \frac{\rho_2}{|m + N\rho|^2} + \text{Oscil.} , \quad (4.1)$$

where τ_2 and ρ_2 are the imaginary parts of τ and ρ , r and q are two integer parameters representing the winding and momentum modes of open strings, respectively. We see that the zero mode part of the spectrum is manifestly invariant under the T-duality group $SO(2, 2; Z) \sim SL(2, Z)_\tau \times SL(2, Z)_\rho$. The other open string parameters are

$$\theta^{01} = -\theta^{10} = 2\pi\alpha'E \quad \theta^{23} = -\theta^{32} = 2\pi\alpha'\frac{B}{1+B^2} , \quad (4.2)$$

$$G_s = g_s \sqrt{(1-E^2)(1+B^2)} . \quad (4.3)$$

Now we take the $E \rightarrow 1$ limit while keeping α', G_s and the volume of the torus fixed. This leads to a NCOS theory on T_θ^2 defined by parameters: $(\alpha', G_s, \tilde{\theta}, \chi; m, N, R_1, R_2)$. The $SL(2, Z)_\tau$ part consists of transformations under which $\tilde{\theta}, m, N, \chi$ and G_s are invariant; it only acts on the torus metric (and r and q modes). Other transformations are generated by $SL(2, Z)_\rho$ matrices which act on the torus volume V, θ, G_s and (m, N) as [1]

$$V' = V (a + b\vartheta)^2 , \quad G'_s = G_s (a + b\vartheta) , \quad \vartheta' = \frac{c + d\vartheta}{a + b\vartheta} , \quad (4.4)$$

$$\begin{pmatrix} m' \\ N' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} m \\ N \end{pmatrix} , \quad ad - bc = 1 , \quad (4.5)$$

where $\vartheta = \frac{\tilde{\theta}}{V}$. Thus, under this transformation, a NCOS theory is mapped into another NCOS theory with the same α' and χ parameters, while all other moduli are transformed as above. For

the special case of rational ϑ there is a T-duality under which ϑ vanishes and the resulting theory is NCOS theory with $\tilde{\theta} = 0$.

Type IIB string theory has in addition the strong-weak $SL(2, Z)$ duality symmetry. This S-duality combined with the above mentioned T-duality group in the usual manner yields the U-duality group $SL(3, Z) \times SL(2, Z)$.

Since general 3+1 NCOS can be obtained from the OM theory [10] on a torus [9], the NCOS on compactified on a T^2 can also be obtained as a limit of OM theory compactified on a 4-torus T^4 . And therefore it may seem that the U-duality symmetry group of NCOS theory could be larger than the $SL(3, Z) \times SL(2, Z)$ group. However, this is not the case, because in order to obtain NCOS theory in 3+1 dimensions (with no additional Kaluza-Klein states coming from $d = 6$) one must take a zero radius limit of R_4, R_5 which breaks the additional symmetry. Indeed, after compactifying M-theory on $T^2 \times T^2$ one has type IIB string theory in $d = 7$, i.e. type IIB on $R^7 \times T^3$, which has a larger U-duality group, $SL(5, Z)$. To have type IIB string theory in $d = 8$, one needs to take an infinite radius limit of one of the S^1 in T^3 (which, in M-theory variables, corresponds to the zero area limit of one of the 2-torus). This reduces the U-duality group to $SL(3, Z) \times SL(2, Z)$.

In the end I would like to mention two interesting open problems:

i) The NCOS strings, being a string theory, show the stringy thermodynamics, in particular the Hagedorn transition [12]. On the other hand, the theories at strong coupling are related to noncommutative gauge theories. It would be quite interesting to see whether this phase transition can also be realized in the field theory limit. There are some motivations that it should. In particular we note that the monopole solutions of noncommutative gauge theories, unlike the commutative case, are not point-like objects and they are stringy. So it seems that in the regime that the dynamics is governed by monopoles, the strong coupling, one should be able to see the stringy behaviour including Hagedorn transition.

ii) The noncommutative open strings we discussed here carry $NCU(N)$ Cahn-Paton factors and hence they are oriented open string theories. It would be very interesting to look for the formulation of *unoriented* noncommutative open strings. Although it may seem problematic at first sight, since NC $SO(N)$ and $Sp(N)$ gauge theories have been formulated [13], the unoriented noncommutative open strings are those appear as the strong coupling limit of these gauge theories [14].

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